Problem. Let $\Omega \subset\subset \mathbb{R}^N$ be a geometrically convex domain, $p \in \partial\Omega$, and $q \in \Omega$. Show that $\forall 0 < t < 1$, we have $(1-t)q + tp \in \Omega$.

Proof. Fix $t \in (0,1)$ and let $x = (1-t)q + tp$. Since $q \in \Omega$, $\exists \epsilon_q > 0 : B_{\epsilon_q}(q) \subset \Omega$. Let $\epsilon_p = (1-t)\epsilon_q > 0$. Since $p \in \partial\Omega$, $\exists z \in \Omega : |z - p| < \epsilon_p$. Let $y = \frac{1}{1-t}(x - tz)$.

Then $|y - q| = \left|\frac{1}{1-t}(x - tz) - \frac{1}{1-t}(x - tp)\right| = \frac{t}{1-t}|p - z| < \frac{t}{1-t}\epsilon_p = \frac{t}{1-t}((1-t)\epsilon_q) = t\epsilon_q < \epsilon_q$ since $0 < t < 1$. Thus, $y \in B_{\epsilon_q}(q) \subset \Omega$, and so $x = (1-t)y + tz \in \Omega$ (since Ω is geometrically convex, and $y, z \in \Omega$). ■